# 6th Form Scholarship Paper 2021 

Date:

Time: 1 hour
Total marks available: 75
Total marks achieved:

Calculator Allowed

## International GCSE Mathematics

## Formulae sheet - Higher Tier

| Arithmetic series Sum to $n$ terms, $S_{n}-\frac{n}{2}[2 a+(n-1) d]$ | Area of trapezium $=\frac{1}{2}(a+b) h$ |
| :---: | :---: |
| The quadratic equation <br> The solutions of $a x^{2}+b x+c-0$ where $a \neq 0$ are given by: $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |  |
| Trigonometry | In any triangle $A B C$ <br> Sine Rule $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ <br> Cosine Rule $a^{2}=b^{2}+c^{2}-2 b c \cos A$ <br> Area of triangle $=\frac{1}{2} a b \sin C$ |
| Volume of cone $-\frac{1}{3} \pi r^{2} h$ <br> Curved surface area of cone $-\pi r l$ | Volume of prism $=\text { area of cross section } \times \text { length }$ |
| Volume of cylinder $=\pi r^{2} h$ Curved surface area of cylinder $-2 \pi r h$ | Volume of sphere $-\frac{4}{3} \pi r^{3}$ <br> Surface area of sphere $-4 \pi r^{2}$ |

Q1.

The integer $N$ is greater than 120
When $N$ is divided by 28 the remainder is 3
When $N$ is divided by 120 the remainder is 3
Find the least value of $N$.
You must show your working.

Q2.

A shopkeeper sells a radio for $\$ 27$
For this selling price, the shopkeeper makes a profit of $8 \%$
Calculate the selling price of the radio so that the shopkeeper would make a profit of $35 \%$
$\qquad$

Q3.

Michael's age is $n$ years.
Navtej's age is three times Michael's age.
Indre is 8 years younger than Navtej and 20 years older than Michael.
Find the value of $n$. Show clear algebraic working.
$n=$
(Total for question = 3 marks)
Q4.

Express as a single fraction in its simplest form

$$
\frac{5}{3-2 x}-\frac{2}{2+5 x}
$$

Q5.

The diagram shows a half cylinder.


Diagram NOT accurately drawn

The half cylinder has a length of $x \mathrm{~cm}$ and a semi-circular cross section of diameter 5 cm .
The volume of the half cylinder is $35 \mathrm{~m} \mathrm{~cm}^{3}$
Calculate the value of $x$.
$\qquad$

Q6.
(a) Solve the inequality $x^{2}-x-6<0$
(b) On the number line below represent your answer to part (a).

|  | 1 | 1 | 1 | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5

(2)
(Total for question = 6 marks)

Q7.

Here is $\triangle A B C$.

(a) Calculate the area, in $\mathrm{cm}^{2}$, of $\triangle A B C$.

Here is $\triangle P Q R$.


Diagram NOT accurately drawn
(b) Calculate the value, to 3 significant figures, of $x$.

Q8.
(a) Simplify fully $\left(5 a^{2} b^{3}\right)^{2}$
(b) Simplify fully $\frac{\left(9 x^{4} y^{2}\right)^{\frac{1}{2}}}{3 x^{2} y^{-4}}$

Q9.

There are 9 marbles in bag $A$ and 11 marbles in bag $B$.
In bag $A$, there are 3 red marbles, 2 yellow marbles and 4 green marbles.
In bag $B$, there are 2 red marbles, 4 yellow marbles and 5 purple marbles.
Ted takes at random one marble from bag $A$ and one marble from bag $B$.
Calculate the probability that the two marbles are not the same colour.

Q10.


Diagram NOT accurately drawn

Figure 1
Figure 1 shows a quadrilateral $A B C D$.
$A B=9.3 \mathrm{~cm} \quad A C=5 \mathrm{~cm} \quad A D=10.8 \mathrm{~cm}$

$$
\angle A C D=124^{\circ} \quad \angle A C B=90^{\circ} \quad \angle C A B=x^{\circ} \quad \angle A D C=y^{\circ}
$$

Calculate to one decimal place,
(a) the value of $x$,
(b) the value of $y$.

Q11.

In a region of a country, two types of eagle, type $A$ and type $B$, can be found.
In 2003 the ratio of the number of type $A$ eagles to the number of type $B$ eagles was $2: 5$ In 2015 the ratio of the number of type $A$ eagles to the number of type $B$ eagles was $4: 3$

From 2003 to 2015, the number of type $A$ eagles had increased by 16
From 2003 to 2015, the number of type $B$ eagles had decreased by 107
Calculate the number of type $B$ eagles in this region in 2015

Q12.

Solve the simultaneous equations

$$
\begin{aligned}
2 x^{2} & =11-3 y^{2} \\
4 x-y & =5
\end{aligned}
$$

Show clear algebraic working.

Q13.

In 2017, country $A$ had a population of $2.35 \times 10^{7}$ people.
Of these people, $48 \%$ were male.
(a) Calculate the number of males in country $A$ in 2017

Country $A$ is divided into three regions. These three regions are called East Region, Central Region and West Region.

In 2017, the ratio of the number of males in the East Region to the number of males in the Central Region to the number of males in the West Region was $5: 3: 2$
(b) Calculate the number of males in the Central Region in 2017

Give your answer in standard form.

In 2017, the number of females in the Central Region was $12.5 \%$ greater than the number of males in the Central Region.
(c) Calculate the number of females in the Central Region in 2017

In 2010, country $B$ had a population of $2.5 \times 10^{7}$ people.
From 2010 to 2014, the population of country $B$ increased by 2.4\%
From 2014 to 2018, the population of country $B$ decreased by $2.4 \%$
(d) Calculate the population of country $B$ in 2018

## Q14.

90 students from a sixth form college were each asked the following question.
"Do you study any of Biology ( $B$ ), Chemistry ( $C$ ) or Physics ( $P$ )?"
Of these 90 students
7 study all three subjects
15 study Biology and Chemistry
20 study Chemistry and Physics
37 study Chemistry
14 study Biology only
15 study Physics only
The number of these students who study Biology and Physics but not Chemistry is three times the number of these students who study none of these three subjects.

Let $x$ be the number of these students who study none of these three subjects.
(a) Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of $x$ where necessary.

(b) Find the value of $x$.
(c) Find
(i) $\mathrm{n}\left(B^{\prime} \cap C\right)$
(ii) $\mathrm{n}(B \cup C \cup P)$
(iii) $\mathrm{n}\left(B \cap C \cap P^{\prime}\right)$

One of the students is to be chosen at random.
Given that this student studies Chemistry,
(d) find the probability that this student also studies Physics.

Q15.


Diagram NOT accurately drawn

The diagram shows a solid right pyramid $A B C D E$ with a square base $A B C D$ on a horizontal plane. The vertex $E$ of the pyramid is vertically above the centre of the base.

$$
B C=34 \mathrm{~m} \quad E A=E B=E C=E D=32.3 \mathrm{~m}
$$

The point $M$ is the midpoint of $C E$ and the point $N$ is the midpoint of $B E$.
Given that $A N+N M=x$ metres
(a) calculate the value, to 2 significant figures, of $x$.
(b) Calculate the size, to the nearest degree, of angle BAN.

Name:

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Q1.

| Q | Working |  |  | Answer | Mark | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 28=2 \times 2 \times 7 \text { or } 4 \times 7 \\ & 120=2 \times 2 \times 2 \times 3 \times 5 \\ & \text { or } 4 \times 30 \text { oe } \end{aligned}$ <br> Or factor trees |  |  |  |  | M1 For prime factorisation of 28 and 120 (may be at ends of a factor tree), must have $2 \times 2 \ldots$ or $4 \times \ldots$ or for multiples of 120 up to at least 840 or for multiples of 28 up to at least 840 |
| $\operatorname{LCM}(28,120)=840$ |  |  |  |  |  | A1 Allow $2 \times 2 \times 2 \times 3 \times 5 \times 7$ |
|  |  |  |  | 843 | 3 | A1ft For adding 3 to their LCM. <br> The M1 must be awarded. An answer with no working gains no marks |
|  |  |  |  |  |  | Total 3 |

Q2.

| Q | Working | Answer | Mark | Notes |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\frac{27}{1.08}$ or $\frac{27}{108} \times 100[=25]$ |  | M1 For a correct method to <br> find the original price. |  |
| $" \frac{27}{108} \times 100 " \times 1.35$ or $\frac{35}{100} \times " 25$ "oe | M1 dep on previous method <br> mark being <br> awarded. For a correct <br> method to increase <br> their original price <br> by $35 \%$ |  |  |  |

Q3.

| Q | Working | Answer | Mark | Notes |
| :--- | :--- | :---: | :---: | :--- |
|  | Sight of $3 n$ or $3 n-8$ or $n+20$ |  | M1 One correct expression seen. May be <br> seen as part of an equation |  |
|  | $n+20=3 n-8$ oe | 14 | 3 | A1 Correct equation |
|  |  | A1 dep on previous A mark awarded |  |  |

Q4.

| Question | Working | Answer | Mark | Notes |  |
| :--- | :--- | :--- | :--- | :---: | :--- |
|  |  | $\frac{5(2+5 x)-2(3-2 x)}{(3-2 x)(2+5 x)}(\mathrm{oe})$ | $\frac{10+25 x-6+4 x}{(3-2 x)(2+5 x)}$ |  | 3 |

Q5.

| Question | Working | Answer | Mark |
| :---: | :---: | :---: | :---: |
|  | $\frac{1}{2} \times \pi \times\left(\frac{5}{2}\right)^{2} \times x(=35 \pi)$ oe |  | 3 |
|  | $x=35 \times\left(\frac{2}{5}\right)^{2} \times 2$ |  |  |
|  |  | 11.2 |  |


| M1 oe |  |
| :--- | :---: |
| M1 dep oe |  |
| A1 oe <br> SC If no marks awarded otherwise 5.6 or 2.8 <br> given as final answer award B1 |  |
|  |  |

Q6.

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Question \& Working \& Answer \& Mark \& AO \& Sub-total \& Total <br>
\hline \multirow[t]{5}{*}{(a)} \& Rewriting (or solving) \& \multirow{7}{*}{$$
\begin{gathered}
x=3 \text { and } x=-2 \\
x<3 \\
x>-2 \\
(\text { or }-2<x<3)
\end{gathered}
$$} \& \& 1.3 \& \multirow[b]{7}{*}{4

2} \& \multirow[b]{7}{*}{6} <br>

\hline \& $$
\begin{aligned}
& x^{2}-x-6<0 \text { as } \\
& (x-3)(x+2) \text { (solving trinomial }
\end{aligned}
$$ \& \& M1 \& \& \& <br>

\hline \& quadratic marking rules) \& \& A1 \& \& \& <br>
\hline \& \& \& A1ft \& \& \& <br>

\hline \& \& \& $$
\begin{aligned}
& \mathrm{A} 1 \mathrm{ft} \\
& (\mathrm{~A} 2 \mathrm{ft})
\end{aligned}
$$ \& \& \& <br>

\hline \multirow[t]{2}{*}{(b)} \& Open circles at ' $x=-2$ and $x=3$ ' \& \& B1 ft \& \& \& <br>

\hline \& Line joining ' $x=-2$ and $x=3$ ' \& \& $$
\mathrm{B} 1 \mathrm{ft}
$$ \& \& \& <br>

\hline
\end{tabular}

Q7.

| Question | Working | Answer | Mark | Notes |  |
| :--- | :--- | :--- | :---: | :---: | :--- |
|  | (a) | $\frac{1}{2}(2)(6) \sin 30^{\circ}$ oe |  |  | M1 |
|  | (b) | $\cos x^{\circ}=\frac{4}{12}$ or <br> $\sin x^{\circ}=\frac{[(\sin 90)] \sqrt{12^{2}-4^{2}}}{12}$ <br> $\tan x^{\circ}=\frac{\sqrt{12^{2}-4^{2}}}{4}$ or $\cos x^{\circ}=$ <br> $\frac{12^{2}+4^{2}-\left(\sqrt{12^{2}-4^{2}}\right)^{2}}{2 \times 12 \times 4}$ |  |  |  |

Q8.

Total 4 marks

Q9.

| Question | Working | Answer | Mark | Notes |
| :--- | :--- | :--- | :--- | :--- | :--- |

Q10.

| Question |  | Working | Answer | Mark |
| :--- | :--- | :--- | :---: | :---: |
| (a) | $\cos x=\frac{5}{9.3}[0.537 \ldots]$ |  | 2 |  |
|  | (b) | $\frac{\sin 124}{10.8}=\frac{\sin y}{5}$ oe |  |  |
|  | $y=\sin ^{-1}\left(\frac{5 \sin 124}{10.8}\right)$ oe |  | 3 |  |
|  |  |  |  |  |
|  |  |  | 22.6 |  |


| Question |  | Notes |
| :--- | :--- | :--- |
| (a) | M1 A fully correct method to find the value of $x$ <br> eg $\sin x=\frac{\sqrt{9.3^{2}-5^{2}}}{9.3}$ or $\tan x=\frac{\sqrt{9.3^{2}-5^{2}}}{5}$ or $90-\sin ^{-1}\left(\frac{5}{9.3}\right)$ or <br> $\cos x=\frac{9.3^{2}+5^{2}-\sqrt{9.3^{2}-5^{2}}}{2 \times 9.3 \times 5}$ |  |
|  | (b) | M1 Awrt to 57.5 |
|  | M1dep on previous method mark being awarded. For rearranging to find a <br> value of $y$ or $\sin y=\frac{5 \times \sin 124}{10.8}$ <br> $y=\sin { }^{-1}($ awrt 0.38$)$ oe method to find the value of $y$ |  |
|  |  | A1 awrt 22.6 |
|  |  | Total 5 mart 0.38 or |

Q11.

| Q | Working | Answer | Mark | Notes |
| :---: | :---: | :---: | :---: | :---: |
|  | $2 x+16$ and $5 x-107$ |  |  | M1 or $X+16$ and $Y-107$ and $5 X=2 Y$ |
|  | $\frac{2 x+16}{4}=\frac{5 x-107}{3} \mathrm{oe}$ |  |  | M1 dep Allow one sign error or $\frac{X+16}{Y-107}=\frac{4}{3}$ or Allow $2 x+16=4 y$ and $5 x-107=3 y$ |
|  | $[x=] 34$ |  |  | M1 dep on both previous Method marks. <br> Using a correct method to solve equation(s) leading to $x=\ldots$ or $y=\ldots$ or $5 x=\ldots$ or $X=\ldots$ or $Y=\ldots$ |
|  | $5 \times " 34$ "-107 |  |  | M1 dep on previous mark. or $3 \times 21$ " |
|  |  | 63 | 5 | A1 Working not required, so correct answer scores full marks (unless from obvious incorrect working) |
|  |  |  |  | Total 5 marks |
| Alternative |  |  |  |  |
|  | $T$ is the total number of eagles in 2003 $t$ is the total number of eagles in 2015 |  |  |  |
|  | $\begin{aligned} & \frac{2}{7} T+16 \text { and } \frac{5}{7} T-107 \text { or } \\ & \frac{4}{7} t-16 \text { and } \frac{3}{7} t+107 \end{aligned}$ |  |  | M1 May be seen as part of a correct equation. |
|  | $\frac{2}{7} T+16=\frac{4}{7} t \text { and } \frac{5}{7} T-107=\frac{3}{7} t \text { oe }$ |  |  | M1 dep for 2 correct equations |
|  | $t=147$ or $T=238$ |  |  | M1 dep on both previous Method marks. <br> Using a correct method to solve equation(s) leading <br> to $T=\ldots$ or $t=$ or $5 T=\ldots$ or $3 t=$ |
|  | $\frac{3}{7} \times " 147 \text { " or } \frac{5}{7} \times " 238^{\prime \prime}-107$ |  |  | M1 dep on previous mark. Allow their 147 or their 238 |
|  |  | 63 |  | A1 Working not required, so correct answer scores full marks (unless from obvious incorrect working) |

Q12.

| Q | Working | Answer | Mark | Notes |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 2 x^{2}=11-3(4 x-5)^{2} \text { or } \\ & 2\left(\frac{5+y}{4}\right)^{2}=11-3 y^{2} \end{aligned}$ |  |  | M1 for correct substitution of the linear equation $4 x-y=5$ into the quadratic equation $2 x^{2}=11-3 y^{2}$ to form an (unsimplified) quadratic equation in either $x$ or $y$. This mark can be implied by the second $M$ mark. |
|  | $2 x^{2}=11-3\left(16 x^{2}-40 x+25\right)$ <br> or $2\left(\frac{25+10 y+y^{2}}{16}\right)=11-3 y^{2}$ |  |  | M1 for correct expansion of either their $(4 x-5)^{2}$ or $\left(\frac{5+y}{4}\right)^{2}$ in correct equation (not dependent on previous M mark) |
|  | $\begin{aligned} & 25 x^{2}-60 x+32[=0] \text { or } \\ & 25 y^{2}+10 y-63[=0] \end{aligned}$ |  |  | A1 for a correct 3 term quadratic in either $x$ or $y$ dep on both previous $M$ marks (oe e.g., $50 x^{2}-120 x+64[=0]$, $50 y^{s}+20 y-126[=0]$ <br> etc. look out for all signs reversed) |
|  | $\begin{aligned} & (5 x-4)(5 x-8)[=0] \text { or } \\ & (5 y-7)(5 y-9)[=0] \end{aligned}$ |  |  | M1 correct method for solving their 3-term quadratic - either by formula, completing the square or factorising. By factorising: brackets must expand to give 2 out of 3 correct terms <br> By formula: correct substitution into fully correct formula (allow 1 sign error). By completing the square: must see $\text { e.g., } 25\left(x-\frac{6}{5}\right)^{2} \pm \ldots[=0]$ |
|  | $\begin{aligned} & 4 \times 0.0 \text { " }-y=5 \text { or } \\ & 4 \times 1.6-y=5 \text { or } \\ & 4 x-(-1.8)=5 \text { or } \\ & 4 x-1.4=5 \text { oe } \end{aligned}$ |  |  | M1indep substituting their two $x$ values into either equation leading to values for $y$ or vice versa (not dependent on any previous M marks) - this mark can be implied by correct values (if no working seen). This mark can be implied by both correct pars of values. |
|  |  | $\begin{gathered} (0.8,-1.8) \\ (1.6,1.4) \end{gathered}$ | 6 | A1 for both correct pairs of $x$ and $y$ values (oe e.g., $x=\frac{4}{5}, y=-\frac{9}{5}$ and $x=\frac{8}{5}, y=\frac{7}{5}$ ) <br> This mark is dependent on all previous marks. <br> Correct answer(s) with no working scores no marks |
|  |  |  |  | Total 6 marks |

Q13.

| Question | Working | Answer | Mark | Notes |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $2.35 \times 10^{7} \times 0.48$ |  | 2 | M1 NB $2.35 \times 10^{7} \times 48 \%$ is not sufficient for this mark unless it leads to the correct answer. |
|  |  | $\begin{aligned} & 1.128 \times 10^{7} \text { or } \\ & 11280000 \end{aligned}$ |  | A1 Allow $1.13 \times 10^{7}$ oe eg $11.3 \times 10^{6}$ |
| (b) | $\left(\frac{11280000 "}{10} \times 3\right)$ |  | 2 | M1 |
|  |  | $3.384 \times 10^{6}$ |  | A1 Allow answers between $3.38 \times 10^{6}$ and $3.39 \times 10^{6}$ inclusive must be in standard form. Allow a final answer of $3.4 \times 10^{6}$ if an acceptable value is seen not written in standard form. |
| (c) | "3 384000 " 1.125 |  | 2 | M1 NB "3 384000 " $\times 112.5 \%$ is not sufficient for this mark unless their answer is equal to their value $\times 1.125$ |
|  |  | $\begin{gathered} 3.807 \times 10^{6} \text { or } \\ 3807000 \end{gathered}$ |  | A1 Allow answers between $3.8 \times 10^{6}$ and $3.814 \times 10^{6}$ inclusive oe |
| (d) | $2.5 \times 10^{7} \times 1.024 \times 0.976$ |  | 2 | M1 |
|  |  | $\begin{gathered} 2.49856 \times 10^{7} \text { or } \\ 24985600 \end{gathered}$ |  | A1 Allow answers between 24986000 and 24990000 inclusive oe ISW rounding. |
|  |  |  |  | Total 8 marks |

Q14.


| Question | Notes |
| :---: | :---: |
| (a) | B3 for all correct entries. <br> B2 for 5, 6 or 7 correct entries. <br> B1 for 3 or 4 correct entries. <br> Allow $24-x$ instead of $3 x$ <br> Allow a sum that gives the required number. eg $20-7$ for the 13 Allow 6 and 18 instead of $x$ and $3 x$ respectively. |
| (b) | M1ft For an equation equivalent to $4 x+66=90$ or $3 x=24-x$ or for the sum of their 8 values from their Venn diagram $=90$ Do not ft if one of the 8 values is blank. <br> SC: Award M1 for an equation equivalent to $\frac{4}{3} x+66=90$ |
|  | A1cao This is not implied by 6 on the diagram. |
| (c) (i) | B1ft "their 9" + "their 13" Condone n("their 22") Do not ft if either required area is blank. Ft their $x$ in part (b) Allow in terms of $x$ |
| (ii) | B1ft 90 - "their part(b)" or 90 - "their $x$ from the diagram" or adding their values in the circles. Condone n ("their 84") Do not ft if any of the required areas are blank. Ft their $x$ in part (b) Allow in terms of $x$ |
| (iii) | B1ft "their 8" Condone n ("their 8") Do not ft if the required area is blank. Ft their $x$ in part (b) Allow in terms of $x$ |
|  | SC : If the numbers are listed rather than added or written as a fraction eg 22/90. Award B0 for the first fully correct list/fraction following through from their diagram. The following B marks can then be awarded for a correct list/fraction following through from their diagram. |
| (d) | B2 Allow awrt 0.54 <br> B1 for $\frac{20}{m}$ where $m \geq 20$ or $\frac{n}{37}$ where $n \leq 37$. This should be their final answer and not be in part of a calculation. Ignore cancelling. SC: B1ft for $\frac{" 20^{\prime \prime}}{m}$ with "their 20 " ie "their 7 " + "their 13 " as long as neither blank where $m \geq 20$ Ignore cancelling. |
|  | Total 10 marks |

Q15.


| (b) | $\begin{aligned} & \frac{\sin (\angle B A N)}{16.15}=\frac{\sin " 58.2 \ldots "}{" 28.9 \ldots "} \\ & \text { or } \cos (\angle B A N) \\ & =\frac{34^{2}+" 28.9 \ldots{ }^{\prime 2}-16.15^{2}}{2(34)(" 28.9 \ldots ")} \end{aligned}$ |  | 2 | M1 correct complete method to find angle $B A N$ <br> Allow this mark for those that consider <br> $\frac{\sin (\angle B N A)}{34}=\frac{\sin " 58.2 \ldots "}{" 28.9 \ldots "}$ which leads to either <br> $\angle B N A=86.546 \ldots$ or $93.453 \ldots$ and considers $\angle B A N=180-" 58.2 \ldots "-\angle B N A$ (if using incorrect value then most likely to see the angle given as $35.21 \ldots$ which scores M1 only) |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 28 |  | A1 (correct working only) - must be using a correct $A N$ and angles from part (a) - awrt 28. <br> (For reference: 28.303196...) |
|  |  |  |  | Total 8 marks |

